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Classical q-deformation of $su(2)$ and $Os(1)$

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Abstract. The generators of the classical $su_q(2)$ and $Os_q(1)$ quantum groups are obtained from the non-deformed ones by means of an appropriate mapping and by imposing the Poisson bracket (PB) relations of the deformed algebras. The classic limit of the q-deformed Lipkin model is obtained from a time-dependent variational principle and its orbits are represented in phase space.

In the present work we obtain the classical q-deformed $Os(1)$ and $su(2)$ algebras from the non-deformed ones by means of an appropriate mapping and by imposing the Poisson bracket (PB) relations of the deformed algebra, following the method of [1]. These relations are compared with the mean-field results discussed in [2] and applied to the semiclassical limit of the $su_q(2)$ Lipkin model.

Let $\bar{\alpha}$, $\bar{\alpha}^*$ be a set of coordinates which obey the PB relations of $Os(1)$:

$$\{\bar{\alpha}, \bar{\alpha}^*\} = -\mathrm{i}\,. \tag{1}$$

Defining $N = \bar{\alpha}\bar{\alpha}^*$, we also have

$$\{N, \bar{\alpha}\} = \mathrm{i}\bar{\alpha} \qquad \{N, \bar{\alpha}^*\} = -\mathrm{i}\bar{\alpha}^*\,. \tag{2}$$

We introduce a mapping that transforms the generators of the above algebra, $\bar{\alpha}$, $\bar{\alpha}^*$ and N, into the generators of the classical q-deformed $su(2)$ algebra, $\mathcal{J}_+$, $\mathcal{J}_-$ and $\mathcal{J}_z$:

$$\mathcal{J}_z = N - j \qquad \mathcal{J}_- = \bar{\alpha} g_-(N) \qquad \mathcal{J}_+ = \bar{\alpha}^* g_+(N)\,. \tag{3}$$

The functions $g_-(N)$ and $g_+(N)$ are determined in such a way that the following PB relations are verified:

$$\{\mathcal{J}_z, \mathcal{J}_\pm\} = \mp\mathrm{i}\mathcal{J}_\pm \tag{4a}$$

$$\{\mathcal{J}_+, \mathcal{J}_-\} = -\mathrm{i}[2\mathcal{J}_z] \tag{4b}$$

where

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}}\,. \tag{5}$$

We obtain

$$N g_-(N) g_+(N) = -\frac{\cosh(\gamma(2N - 2j))}{2\gamma \sinh\gamma} + C \qquad \gamma = \ln q\,. \tag{6}$$

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The choice $C = \cosh(\gamma(2j))/(2\gamma \sinh\gamma)$ leads to

$$Ng_-(N)g_+(N) = \frac{\sinh\gamma}{\gamma}[2j - N][N]. \qquad (7)$$

This preserves the appropriate form in the limit $q \to 1$. In the standard representation the generators $\mathcal{J}_-$ and $\mathcal{J}_+$ are the complex conjugate of each other and we get

$$g_+(N) = g_-(N) = \eta\sqrt{[2j - N][N]/N}$$

with

$$\eta = \sqrt{\sinh\gamma/\gamma}.$$

The following representation for the generators of the classical $su_q(2)$ deformed algebra is obtained:

$$\mathcal{J}_z = N - j \qquad \mathcal{J}_- = \bar{\alpha}\eta\sqrt{[2j - N][N]/N} \qquad \mathcal{J}_+ = \bar{\alpha}^*\eta\sqrt{[2j - N][N]/N}. \qquad (8)$$

We also wish to define the generators of the $su_q(2)$ deformed algebra in terms of the generators of the $Os_q(1)$ deformed algebra, $\tilde{\alpha}$, $\tilde{\alpha}^*$ and $\mathcal{N}$. The same procedure will be used but first the $Os_q(1)$ algebra will be determined from the non-deformed $Os(1)$ algebra. Let

$$\mathcal{N} = N \qquad \tilde{\alpha}^* = \bar{\alpha}^* h_+(N) \qquad \tilde{\alpha} = \bar{\alpha} h_-(N). \qquad (9)$$

We choose the functions $h_+(N)$, $h_-(N)$ in such a way that the following PB relations are verified [3]:

$$\{\mathcal{N}, \tilde{\alpha}\} = \mathrm{i}\tilde{\alpha} \qquad (10a)$$

$$\{\mathcal{N}, \tilde{\alpha}^*\} = -\mathrm{i}\tilde{\alpha}^* \qquad (10b)$$

$$\{\tilde{\alpha}, \tilde{\alpha}^*\} = -\mathrm{i}\eta\frac{\mathrm{d}[N]}{\mathrm{d}N}. \qquad (10c)$$

Then, (10a) and (10b) are automatically satisfied and from (10c) we get

$$h_+(N)\, h_-(N) = \eta\frac{[N]}{N}. \qquad (11)$$

In the standard representation, the q-deformed variables $\tilde{\alpha}$, $\tilde{\alpha}^*$ are given by

$$\tilde{\alpha}^* = \bar{\alpha}^*\sqrt{\eta\frac{[N]}{N}} \qquad (12a)$$

$$\tilde{\alpha} = \bar{\alpha}\sqrt{\eta\frac{[N]}{N}}. \qquad (12b)$$

In order to obtain the mapping which expresses the generators of the $su_q(2)$ algebra in terms of the generators of the $Os_q(1)$ algebra we introduce the generators of the $su_q(2)$ algebra by

$$\mathcal{J}_z = N - j \qquad \mathcal{J}_- = \tilde{\alpha} f_-(N) \qquad \mathcal{J}_+ = \tilde{\alpha}^* f_+(N) \qquad (13)$$

where the functions $f_+(N)$, $f_-(N)$ are determined in such a way that the PB relations defined in (4) are verified:

$$f_+(N)\, f_-(N) = \eta[2j - N]\,. \tag{14}$$

In the standard representation,

$$f_-(N) = f_+(N) = \sqrt{\eta[2j - N]}$$

and

$$\mathcal{J}_z = N - j \qquad \mathcal{J}_- = \tilde{\alpha}\sqrt{\eta[2j - N]} \qquad \mathcal{J}_+ = \tilde{\alpha}^*\sqrt{\eta[2j - N]}\,. \tag{15}$$

The last relations resemble the mapping obtained in [1] and [4] except for the appearance of the factor η.

In [2], the $su_q(2)$ quantum algebra is discussed in the framework of the mean-field approximation. Following this reference we consider the coherent state

$$|\Psi\rangle = \sum_n \frac{\alpha^n J_+^n}{[n]!}|\Phi\rangle \tag{16}$$

where

$$J_-|\Phi\rangle = 0 \qquad J_z|\Phi\rangle = -j|\Phi\rangle\,.$$

We introduce the operator $\hat{N}$ such that

$$\hat{N} J_+^n|\Phi\rangle = nJ_+^n|\Phi\rangle$$

and denote the mean value of $[\hat{N}]$ and $[2j - \hat{N}]$ by

$$\langle[\hat{N}]\rangle = \frac{\langle\Psi|[\hat{N}]|\Psi\rangle}{\langle\Psi|\Psi\rangle} \tag{17}$$

$$\langle[2j - \hat{N}]\rangle = \frac{\langle\Psi|[2j - \hat{N}]|\Psi\rangle}{\langle\Psi|\Psi\rangle}\,. \tag{18}$$

The following relation is easily derived:

$$\langle[\hat{N}]\rangle = \alpha^*\alpha\langle[2j - \hat{N}]\rangle\,. \tag{19}$$

In [2], the approximation

$$\langle[\hat{N}]\rangle = \eta[\langle\hat{N}\rangle] \tag{20a}$$

$$\langle[2j - \hat{N}]\rangle = \eta[2j - \langle\hat{N}\rangle] \tag{20b}$$

is proposed. The mean-field values of the generators J_z, J_-, J_+ are

$$\mathcal{J}_z = \frac{\langle\Psi|J_z|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \langle\hat{N}\rangle - j \tag{21a}$$

$$\mathcal{J}_- = \frac{\langle\Psi|J_-|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \alpha\langle[2j - \hat{N}]\rangle \tag{21b}$$

$$\mathcal{J}_+ = \frac{\langle\Psi|J_+|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \alpha^*\langle[2j - \hat{N}]\rangle\,. \tag{21c}$$

Table 1. Values of $C(j, \gamma)$ as a function γ and j.

$C(j,\gamma)$	$j=5$	$j=15$	$j=25$	$j=50$	$j=100$	η
$\gamma=0.05$	1.0030665	1.0079715	1.0106602	1.0124089	1.0125884	1.0002083
$\gamma=0.1$	1.0116237	1.0228884	1.0249722	1.0253125	1.0253153	1.0008334
$\gamma=0.2$	1.0388417	1.0510120	1.0512660	1.0512705	1.0512715	1.0033344

In terms of the approximation introduced in (20), equations (21) take the form

$$\mathcal{J}_z = \langle\hat{N}\rangle - j \tag{22a}$$

$$\mathcal{J}_- = \alpha\eta[2j - \langle\hat{N}\rangle] \tag{22b}$$

$$\mathcal{J}_+ = \alpha^*\eta[2j - \langle\hat{N}\rangle]\,. \tag{22c}$$

From (19), (20) and (12) the following relations between the variables α, α^* and the variables which have been identified as the generators of the classical $Os_q(1)$ algebra, $\tilde{\alpha}$, $\tilde{\alpha}^*$, are obtained:

$$\tilde{\alpha}\tilde{\alpha}^* = \eta[\langle\hat{N}\rangle] = \alpha\alpha^*\eta[2j - \langle\hat{N}\rangle] \tag{23}$$

or

$$\tilde{\alpha}^* = \alpha^*\sqrt{\eta[2j - \langle\hat{N}\rangle]} \qquad \tilde{\alpha} = \alpha\sqrt{\eta[2j - \langle\hat{N}\rangle]}\,.$$

Finally, in the present approximation we get

$$\mathcal{J}_z = \langle\hat{N}\rangle - j \tag{24a}$$

$$\mathcal{J}_- = \tilde{\alpha}\sqrt{\eta[2j - \langle\hat{N}\rangle]} \tag{24b}$$

$$\mathcal{J}_+ = \tilde{\alpha}^*\sqrt{\eta[2j - \langle\hat{N}\rangle]}\,. \tag{24c}$$

With (24) we recover (15).

We recall that the aim of postulating (20*a*, *b*) was to preserve the $su_q(2)$ algebra when the commutators are replaced by PB relations. Actually the numerical results reported in [2] suggest that a slightly improved approximation may be more adequate. This is indeed the case, as we shall see. We conclude that the expressions (24) coincide with (15) previously obtained if $\langle\hat{N}\rangle$ is substituted by the generator $\mathcal{N}$.

In order to test the validity of the approximation imposed by (20) we have calculated the ratios

$$u_1 = \frac{\langle[\hat{N}]\rangle}{[\langle\hat{N}\rangle]} \qquad u_2 = \frac{\langle[2j - \hat{N}]\rangle}{[2j - \langle\hat{N}\rangle]} \tag{25}$$

for several values of j and γ. For $\mathcal{J}_z = \pm j$, the values of the ratios are independent of j and γ: $u_1 = \eta^2$, $u_2 = 1$ for $\mathcal{J}_z = -j$ and $u_1 = 1$, $u_2 = \eta^2$ for $\mathcal{J}_z = j$. We denote by $C(j, \gamma)$ the common value of u_1 and u_2 for $\mathcal{J}_z = 0$.

In table 1 the values of $C(j, \gamma)$ are given as a function of γ and j. The last column of this table gives η for the values of γ considered. We conclude that for large values of j, $C(j, \gamma)$ is a function of γ only. The stabilization of the $C(j, \gamma)$ with j for a given γ is faster the larger is γ. For large j and γ the approximation (20) is indeed improved if η is substituted by $C(\gamma) = C(j = \infty, \gamma)$. It was shown in [2] that u_1, u_2 are approximatly equal for a large range of $\mathcal{J}_z$.

Table 2.

σ	$j = 15$	$j = 25$	$j = 50$
$\gamma = 0$	0.033	0.020	0.005
$\gamma = 0.2$	0.011	0.004	0.001
$\gamma = 0.4$	0.005	0.002	0.0005

We discuss now a possible explanation for this behaviour. We introduce the mean value of $\hat{N}$ using the q-deformed binomial distribution

$$\bar{n} = \frac{\sum_{n=0}^{2j} n \binom{2j}{n}_q}{\sum_{n=0}^{2j} \binom{2j}{n}_q}$$

with

$$\binom{2j}{n}_q = \frac{[2j]!}{[n]![2j-n]!}\,.$$

This corresponds to the expectation value of the operator $\hat{N}$ using the coherent state (16) with $\alpha = 1$. Calculating the mean square root of the variance of n,

$$\sigma^2 = \frac{\overline{n^2} - \bar{n}^2}{\bar{n}^2}$$

for different values of γ we conclude that the value of σ for a given j gets smaller as γ increases. This is seen in table 2.

The numerical results in table 1 suggest an improved version of the approximation (20) leading to

$$\mathcal{J}_z = -j + N \tag{26a}$$

$$\mathcal{J}_- = \tilde{\alpha}\sqrt{\mathcal{C}[2j - N]} \tag{26b}$$

$$\mathcal{J}_+ = \tilde{\alpha}^*\sqrt{\mathcal{C}[2j - N]} \tag{26c}$$

with $N = \langle\hat{N}\rangle$. The difference between (15) and (26b), (26c) lies in the replacement of η by $\mathcal{C} = \mathcal{C}(j, \gamma)$. The new generators of $su_q(2)$ satisfy the PB relations (4a). The PB relation (4b) is slightly modified:

$$\{\mathcal{J}_+, \mathcal{J}_-\} = -\mathrm{i}\left(\frac{\mathcal{C}}{\eta}\right)[2\mathcal{J}_z]\,. \tag{27a}$$

In the present approximation the Casimir operator is given by [2]

$$C_q = \mathcal{J}_+\mathcal{J}_- + \mathcal{C}^2[\mathcal{J}_z]^2\,. \tag{27b}$$

We will study the classical limit of the deformed Lipkin model [5]. The model consists of N fermions interacting via one- and two-body forces. The fermions are distributed in two levels, each having an N-fold degeneracy. The energy difference between the levels is ϵ.

Owing to the symmetries of the system, the Hamiltonian of the system can be written in terms of the generators of $su(2)$:

$$H = \epsilon J_z + \frac{V}{2}(J_+^2 + J_-^2)\,. \tag{28}$$

In the sequel we will consider that the energy of the system is obtained from the above Hamiltonian but the generators J_z, J_+, J_- obey the commutation relations of the $su_q(2)$ algebra [6]. The ground state of the system if $V = 0$ is the state with all the particles in the lowest energy level and $\mathcal{J}_z = -j$:

$$|0\rangle = |j\,-j\rangle\,.$$

For $V \neq 0$ we use the coherent state introduced in (16) to describe the system. We are replacing the total Hilbert space of the quantum system by the manifold spanned by the variables α, α^*.. The expectation value of a general operator $\hat{X}$ is given by

$$\langle\hat{X}\rangle = \frac{\langle\Psi|\hat{X}|\Psi\rangle}{\langle\Psi|\Psi\rangle}\,. \tag{29}$$

The dynamics of the system will be described by the time evolution of the variables α, α^*. The action in terms of these coordinates is given by

$$S = \int\left(\mathrm{i}\frac{\langle\Psi|\dot{\Psi}\rangle - \langle\dot{\Psi}|\Psi\rangle}{2\langle\Psi|\Psi\rangle} - \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)\mathrm{d}t = \int\left(\mathrm{i}\frac{\dot{\alpha}\alpha^* - \alpha\dot{\alpha}^*}{2\alpha\alpha^*}\langle\hat{N}\rangle - \mathcal{H}(\alpha,\alpha^*)\right)\mathrm{d}t \tag{30}$$

with

$$\mathcal{H}(\alpha,\alpha^*) = \epsilon\left(-j + \langle\hat{N}\rangle\right) + \frac{V}{2}(\alpha^2 + \alpha^{*2})\langle[2j - \hat{N}][2j - \hat{N} - 1]\rangle\,. \tag{31}$$

In terms of the expectation values of the operators J_z, J_+, J_- already calculated in (21) we have

$$\mathcal{H} = \epsilon\mathcal{J}_z + \frac{V}{2}(\mathcal{J}_+^2 + \mathcal{J}_-^2)\frac{\langle[j - J_z][j - J_z - 1]\rangle}{\langle[j - J_z]\rangle^2}\,. \tag{32}$$

We may say that the factor $\langle[j - J_z][j - J_z - 1]\rangle/\langle[j - J_z]\rangle^2$ accounts for Pauli principle effects. If this factor is replaced by 1 the exchange effects are neglected. In the limit $q \to 1$ this last expression reduces to [7]

$$\mathcal{H} = \epsilon\mathcal{J}_z + \frac{V}{2}\frac{2j - 1}{2j}(\mathcal{J}_+^2 + \mathcal{J}_-^2)\,. \tag{32a}$$

We define the classical limit of the present model as the system whose dynamics is described by the Hamiltonian (28) with the operators J_z, J_+, J_- substituted by the classical generators defined in (26). This corresponds to the Hamiltonian (32) excluding the Pauli principle effects. Instead of $\mathcal{J}_+$, $\mathcal{J}_-$ we could have chosen $\mathcal{J}_x$, $\mathcal{J}_y$. For a given energy E the trajectory of the system is given by the intersection of two surfaces; one defines E:

$$E = \epsilon\mathcal{J}_z + V(\mathcal{J}_x^2 - \mathcal{J}_y^2) \tag{33a}$$

and the other, obtained from (27*b*), the total angular momentum:

$$\mathcal{J}_x^2 + \mathcal{J}_y^2 + \mathcal{C}^2[\mathcal{J}_z]^2 = \mathcal{C}^2[j]^2 . \tag{33b}$$

There are two possible types of motion according to the value of the parameter

$$\chi = \frac{[2j]}{\epsilon}|V| . \tag{34}$$

(i) For $\chi < 1$ the ground-state energy is

$$E_0 = -\epsilon j \tag{35}$$

and the $su(2)$ generators have the values

$$\mathcal{J}_z = -j \qquad \mathcal{J}_\pm = 0 . \tag{36}$$

The energy of the normal mode of the system in this regime is

$$\Omega = \sqrt{1-\chi^2} . \tag{37}$$

It is clearly seen from the normal mode energy that $\chi = 1$ defines a phase transition to a new regime. We point out that the parameter which defines this phase transition is no longer $2j|V|/\epsilon$ as in the $q = 1$ limit.

(ii) For $\chi > 1$ the ground-state energy corresponds to a value of $\mathcal{J}_x$ different from zero and $\mathcal{J}_z > -j$.

If the ground-state energy is calculated from (32) without further approximations we obtain the same two different regimes. The parameter which defines the phase transition in the present case is

$$\chi' = \frac{[2j-1]}{\epsilon}|V| . \tag{38}$$

For $\chi' < 1$ the energy of the ground state, $\mathcal{J}_z$, $\mathcal{J}_\pm$ and the energy of the normal mode are given by (35), (36) and (37) respectively, with χ substituted by χ'.

In the sequel we choose $\epsilon = 0.1$ MeV. In figures 1(*a*) and (*b*) we compare, for two different values of j ($j = 15$ and 50) and $\gamma = 0.05$ and 0.2, the exact Lipkin ground-state energy (triangles) with the classical one (broken curve). The ground-state energies are plotted as a function of the parameter χ'. The classical ground-state energies were obtained minimizing (33*a*) with respect to $\tilde{\alpha}$ and $\tilde{\alpha}^*$. For $j = 15$ the agreement improves with γ as expected. The slope of the curve representing the classical results for $\gamma = 0.05$ is bigger because the Pauli principle is not taken into account.

In figures 2(*a*) and (*b*) we represent in the plane $\mathcal{J}_x\mathcal{J}_y$ the orbits of the system for several energies. We have chosen $\chi = 5.0$, $j = 15$ and $\gamma = 0.0$ (figure 2(*a*)) and $\gamma = 0.2$ (figure 2(*b*)). The phase-space trajectories lie on a sphere of radius j in the case $\gamma = 0.0$ or on an ellipsoidal surface for $\gamma = 0.2$ and correspond to the intersection of these geometric figures with the energy surface (33*a*), a parabolic–hyperbolic surface. In both figures the full triangles correspond to the two energy minima (E_0) of the system and the open circles to the two energy maxima ($E_{max} = -E_0$). For energies close to E_0 ($-E_0$) the trajectory will describe a closed path around E_0 ($-E_0$). In this case there are two possible trajectories for each energy. This occurs for $\gamma = 0.0$ at $E = 0.95E_0$ and $0.4E_0$ ($E = -0.95E_0, -0.4E_0$). For $\gamma = 0.2$ and

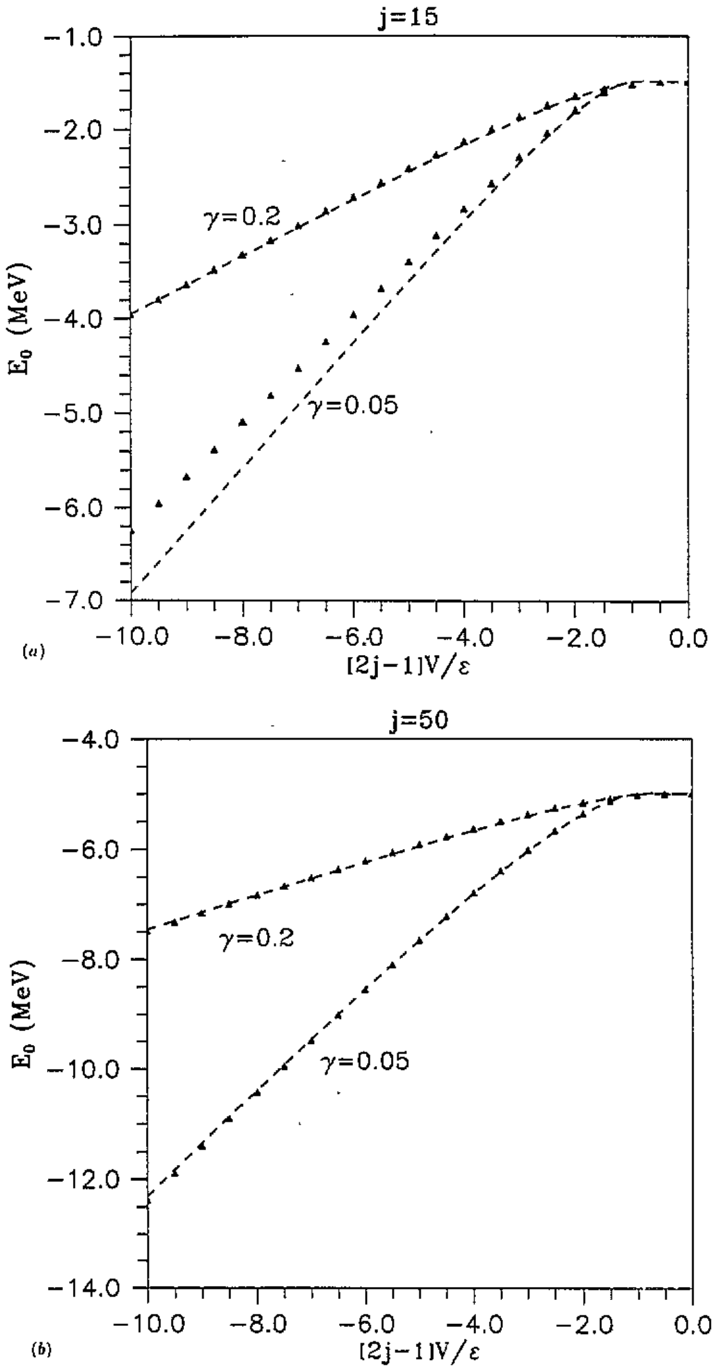
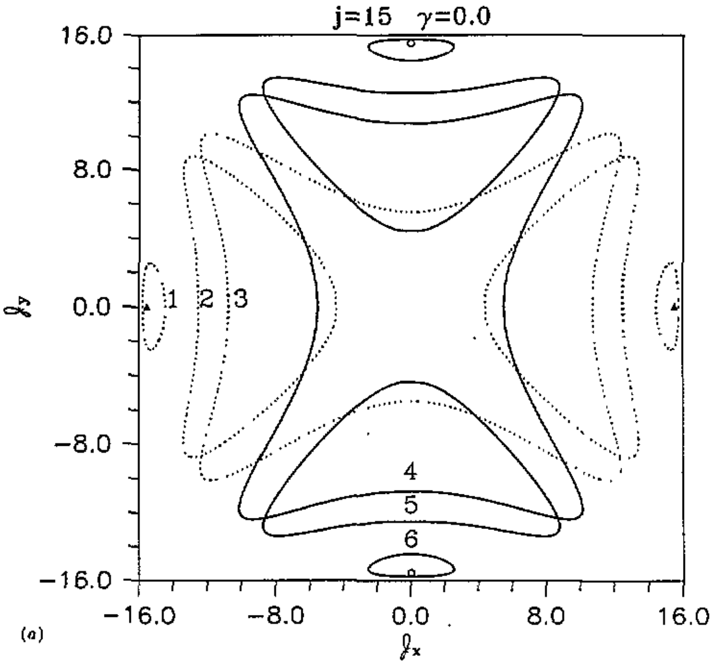


Figure 1. (*a*) The exact (full triangles) and classical (broken curve) ground-state energies as a function of χ', for $j = 15$. (*b*) The exact (full triangles) and classical (broken curve) ground-state energies as a function of χ', for $j = 50$.

$E = 0.4E_0$ ($E = -0.4E_0$) the trajectory turns around $\mathcal{J}_z$, a behaviour which occurs for all values of the energy if $\chi < 1$. For $\chi > 1$, the transition to this regime occurs when $E > -\epsilon j$



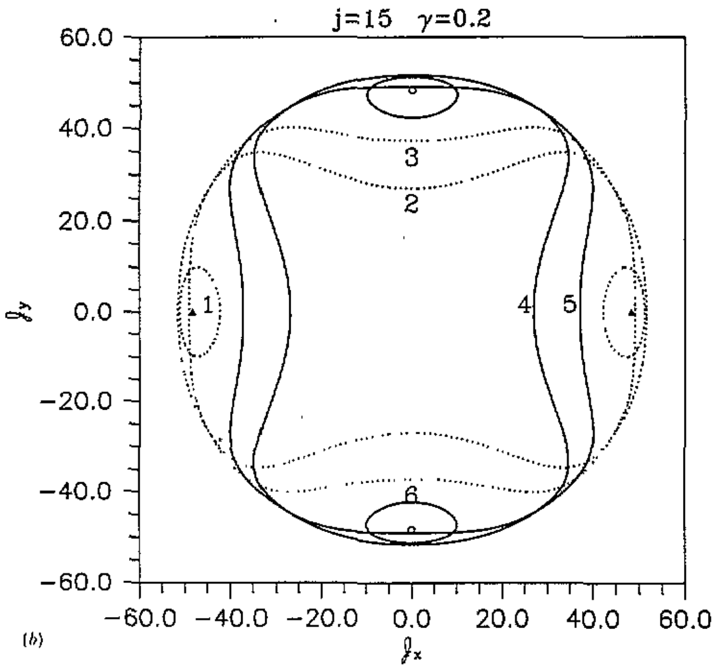


Figure 2. The trajectories for the classical Lipkin model in the $\mathcal{J}_x\mathcal{J}_y$ plane for $\chi = 5.0$. The static solutions are represent by the full triangles (minima, $E = E_0$) and open circles (maxima, $E = -E_0$). The labels 1–6 correspond respectively to trajectories with energies $0.95E_0$, $0.4E_0$, $0.2E_0$, $-0.2E_0$, $-0.4E_0$, $-0.95E_0$ ($E_0 = -4.42$ MeV). (*b*) As in (*a*) for $\gamma = 0.2$, $E_0 = -2.5$ MeV.

or $E < \epsilon j$. At $E = -\epsilon j$ and $E = \epsilon j$ the energy parabola vertex crosses towards the south or north pole, respectively, of the ellipsoidal surface (33*b*). The curvature of the energy surface defines the ground-state energy and the behaviour of the phase-space trajectories. On the $\mathcal{J}_z\mathcal{J}_y$ plane the curvature is equal to $2\chi/[2j]$, and therefore decreases with γ. This explains the increase of the ground-state energy with γ for the same χ: the intersection of the two surfaces (33) is only possible at higher energies.

We conclude that for a fixed value of the parameter χ the deformed phase solution gets closer to the normal phase solution when γ increases. Similar conclusions have been reached in [6]. We would like to point out, however, that in order to keep χ constant when the parameter γ increases the two-body interaction strength V decreases drastically.

We recover the quantum description of the system choosing the allowed orbits as the ones which verify the Sommerfeld relation

$$\oint p\,\mathrm{d}q = 2\pi\left(n + \tfrac{1}{2}\right)$$

or, in terms of $\mathcal{J}_x, \mathcal{J}_y$,

$$\iint \frac{\mathrm{d}\mathcal{J}_x \mathrm{d}\mathcal{J}_y}{\{\mathcal{J}_x, \mathcal{J}_y\}} = 2\pi\left(n + \tfrac{1}{2}\right).$$

We have obtained a realization of the classical $su_q(2)$ in terms of the generators of both the classical $Os(1)$ and $Os_q(1)$ algebras. This is compared with the mean-field results obtained in [2] for the generators of $su_q(2)$. Our expressions coincide with the mean-field values if the classical generator $\mathcal{N}$ is substituted by the mean value of the number operator $\langle\hat{N}\rangle$.

From a numerical analysis of the validity of the mean-field approximation it was shown that a small improvement could be introduced in the definition of the algebra generators. According to this the new generators are given by (26).

The q-deformed Lipkin model is studied in the classical approximation. It is shown that the model parameter is $\chi' = [2j-1]|V|/\epsilon$ in the mean-field approximation, and $\chi = [2j]|V|/\epsilon$ in the classical approximation (Pauli principle effects are neglected). The transition from the normal phase to the deformed one occurs when the model parameter is equal to 1, in both approximations. For a given j we have verified that the classical approximation improves with the deformation parameter γ. Finally we have proposed the Sommerfeld relation as a way of recovering the quantum description.

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